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Development of a novel variable rake angle mechanism for noncircular turning and its control †

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Abstract

A prototype tool stage with a variable rake angle mechanism has been developed and controlled. The objective of the variable rake angle mechanism is to provide two-degrees-of-freedom to the conventional noncircular turning process, so that the rotational tool mechanism can compensate for the rake angle change caused by the noncircular cam profile itself while the translational tool motion generates the cam profile. Inverse kinematics, kinematic sensitivity and stiffness, and design of the variable rake angle mechanism are discussed. Robust repetitive controllers are designed for two actuators in the variable rake angle mechanism. Two actuators operate together to change the tool position as well as the tool angle. Experimental results on the variable rake angle mechanism support the design concept and control approach.

Keywords: Noncircular turning; Repetitive control; Robust control; Variable rake angle mechanism

1. Introduction

There are many cases in the industrial control area where the reference and/or disturbance signals are periodic. To utilize this characteristic of the periodic signal in control system design, a variety of repetitive controllers has been developed and applied to several applications. One of the manufacturing applications is the noncircular turning process. Noncircular turning generates a workpiece with noncircular cross section by direct tool motion control. The noncircular turning process, especially camshaft machining and piston turning have been studied by several researchers [1-4], but they used only one actuator to generate noncircular robes in their work. For cam turning, since a cutting tool approaches a workpiece in radial direction, the rake angle between a tool bit and a workpiece changes according to the cam profiles. For example, Fig. 1 shows a cam profile with radial and normal vectors at each point of its circumference. If we assume that the cutting tool has zero rake angle in itself and it can't change its angular position with respect to the workpiece, then the rake angle is zero when the cutting tool cuts the bottom half of the cam because the radial direction coincides with the normal direction defined at the same point in this area. The rake angle becomes negative for the upper left part and then, it turns positive for the upper right part when the workpiece rotates clockwise during machining process as shown in Fig. 1. In this particular cam profile, the maximum rake angle change due to the cam profile itself is 25.8 degrees and this amount may be enough to cause tool breakage or rough final finish.

The objective of a variable rake mechanism is to provide two-degrees-of-freedom to the cutting tool by adding another actuator motion, such that it can compensate for the rake angle change or keep a certain degree of rake angle in the noncircular machining process. A prototype of the variable rake angle mechanism has been built in this research. The mechanism structure and experimental results are presented in the following sections.

The remainder of this paper is organized as follows: first, mechanism design is introduced. Second, inverse kinematics, kinematic sensitivity and stiffness of the variable rake angle mechanism are presented. Finally, a robust repetitive control system for the rake angle mechanism is presented. Experimental results are given to illustrate the tracking performance of the prototype variable rake angle mechanism and to validate the mechanism design concept.

2. Design of the variable rake angle mechanism

The mechanism structure and its equivalent linkage are shown in Fig. 2. The tip of tool bit is located at the center of the circular tool slide (labeled with *a*), therefore tool position

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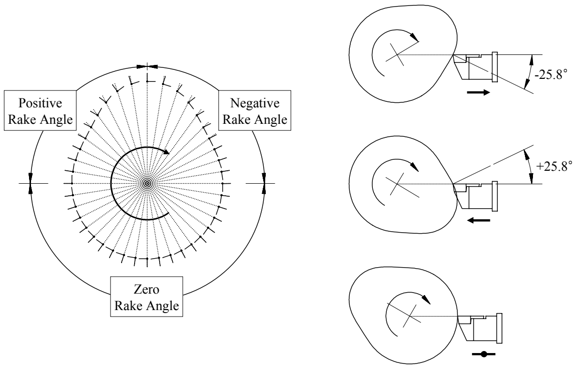


Fig. 1. Rake angle change on a cam profile.

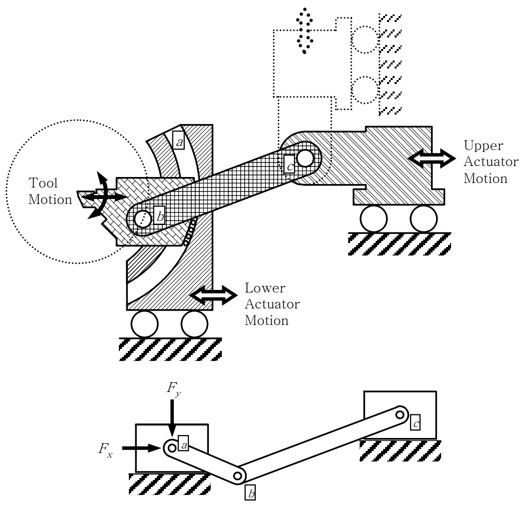


Fig. 2. Schematic of the mechanism and its equivalent linkage.

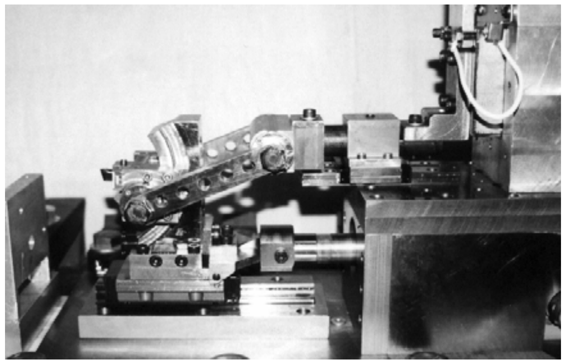
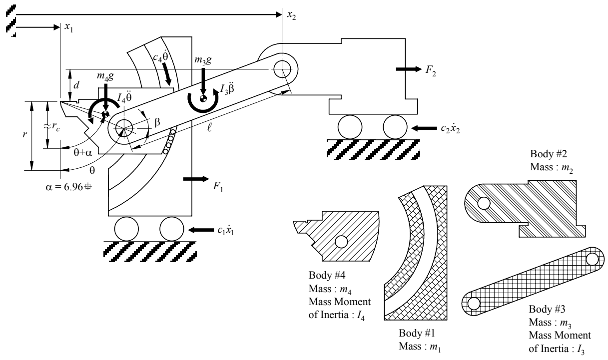


Fig. 3. Photograph of the variable rake angle mechanism.



m_1 (kg)	m_2 (kg)	m_3 (kg)	m_4 (kg)	I_3 (kg·mm^2)	I_4 (kg·mm^2)	r_c (mm)	r (mm)	ℓ (mm)	d (mm)
2.870	1.934	0.182×2	0.953	223.85×2	322.31	45.5	43.1	105.0	20.0

Fig. 4. Design variables of the variable rake angle mechanism.

is independent of the change in tool angle. In the figure, we can see that tool position is governed only by the lower actuator motion and tool angle is determined by the relative motion between the lower and upper actuators. This is an advantage of this structure because the kinematics of the upper actuator has nothing to do with the most important tool position.

In the figure of the equivalent linkage, there are two sliding blocks representing the pistons of the lower and upper actuators, two linkage bars $\overline{ab}$ and $\overline{bc}$, and three pivots a, b and c. The pivot a is functionally equivalent to the circular tool slide. If we assume that cutting force is a point force acting at the end of tool tip and cutting process generates negligible moments, the components of cutting force will appear as F_x and F_y in Fig. 2. From this equivalent linkage configuration, it is obvious that main part of the cutting force should be sustained by the lower actuator.

When the same movement is generated by two actuators, there is no tool angle change. In order to change the tool angle as well as tool position, the upper actuator should have an additional movement for tool angle change on top of the lower actuator movement. Therefore, even though cutting force has little effect on the upper actuator, the upper actuator should be able to produce enough acceleration. An electrohydraulic actuator is used for the lower actuator because the lower actuator is required to have power to support the main block in the mechanism and sustain the cutting force. A voice coil actuator is adopted for the upper actuator because we don't need strong force from the upper actuator but agility. The designed variable rake mechanism is shown in the photograph of Fig. 3.

In Fig. 4, the coordinate system of the variable rake angle mechanism is described in terms of the coordinates x_1 and x_2, which describe the horizontal positions of the lower and upper actuators, respectively. The mass of the main body (Body #1) supporting the rotary tool part is m_1, and m_2 is the mass of sliding body (Body #2) connected to the upper actuator. m_3 is the mass of a pair of the connecting rods (Body #3) and m_4 is the mass of the rotary tool part (Body #4). c_1, c_2, and c_4 are viscous damping coefficients. I_3 and I_4 are mass moments of inertia about mass center of the connecting rods and the rotary tool part, respectively. The forces from the lower and upper actuators are represented by F_1 and F_2. In the Figure, α is 6.96 degrees and the accompanying table shows other design variables computed from the prototype design.

3. Kinematics

The inverse kinematics problem is defined as computing the actuator displacements, when the end-effector position and orientation are given. Let the tool angle be defined as the angle of the rotary cutting tool part starting from the configuration depicted in Fig. 4, i.e. tool angle = θ - 67.18 degrees. The desired end-effector posture in the variable rake angle mechanism is described by the tool position and tool angle. These quantities are represented with x_1 and θ. The actuator displacements are described by x_1 and x_2. Thus, only x_2 needs to be formulated in the inverse kinematics problem. By observation of geometric relationships in Fig. 4, the inverse kinematics is formed as follows:

$$x_2 = x_1 + r\sin\theta + \sqrt{\ell^2 - (d + r\cos\theta)^2} \tag{1}$$

When a cam profile is given, x_1 is equal to the cam lift and x_2 is calculated by substituting x_1 and θ into Eq. (1). With a given spindle speed, velocity and acceleration can be obtained by taking first and second time derivatives of displacement.

Let $\Delta\mathbf{q} = [\Delta x_1, \Delta\theta]^T$ be the vector characterizing the change in the end-effector posture, and $\Delta\mathbf{p} = [\Delta x_1, \Delta x_2]^T$ be the vector characterizing the change in the actuator displacements. The kinematic sensitivity is as follows:

$$\Delta\mathbf{p} = \begin{bmatrix} \dfrac{\partial x_1}{\partial x_1} & \dfrac{\partial x_1}{\partial\theta} \\ \dfrac{\partial x_2}{\partial x_1} & \dfrac{\partial x_2}{\partial\theta} \end{bmatrix} \Delta\mathbf{q} = \begin{bmatrix} 1 & 0 \\ 1 & \xi \end{bmatrix} \Delta\mathbf{q} = \mathbf{J}\Delta\mathbf{q}, \tag{2}$$

where

$$\xi = r\cos\theta + \frac{r\sin\theta(d + r\cos\theta)}{\sqrt{\ell^2 - (d + r\cos\theta)^2}}. \tag{3}$$

The matrix $\mathbf{J}$ is called the Jacobian matrix and represents kinematic sensitivities between the actuator displacements and the end-effector posture. A point referred as configuration singularity makes ξ be equal to zero. One nontrivial singular point of the variable rake angle mechanism is a toggle position. It is described as

$$\cos\theta = -\frac{d}{r + \ell} \quad \Rightarrow \quad \xi = 0. \tag{4}$$

With the design parameters shown in Fig. 4, the singular point is computed as θ = 97.76 degrees and this position should be avoided during mechanism operation. The sensitivity from $\Delta\theta$ to Δx_2 which is denoted by ξ also can be computed from Eq. (3). When the tool angle is zero (or θ = 67.18 degrees), the upper actuator needs to move 0.55 mm for 1 degree change in the tool angle.

Let $\mathbf{t} = [F_x, M_z]^T$ be the end-effector wrench vector, where F_x is a x-directional force and M_z is a counterclockwise moment acting at the tip of the tool bit. The wrench applied to the end-effector is related to its displacement twist by [5]:

$$\mathbf{t} = \mathbf{J}^T\mathbf{K}\Delta\mathbf{p} = \mathbf{J}^T\mathbf{K}\mathbf{J}\Delta\mathbf{q} = \begin{bmatrix} k_1 + k_2 & \xi k_2 \\ \xi k_2 & \xi^2 k_2 \end{bmatrix} \Delta\mathbf{q}, \tag{5}$$

where $\mathbf{K} = diag\ (k_1, k_2)$, k_1 and k_2 are stiffnesses of the upper and lower actuators, respectively. The maximum and minimum stiffnesses of the variable rake angle mechanism are equal to the maximum and minimum eigenvalues γ_{max} and γ_{min} of the $\mathbf{J}^T\mathbf{K}\mathbf{J}$ matrix, respectively. These bounds are computed by

$$\gamma_{\max}, \gamma_{\min} = \frac{1}{2}\Big(k_1 + k_2 + \xi^2 k_2 \pm \sqrt{-4\xi^2 k_1 k_2 + (k_1 + k_2 + \xi^2 k_2)^2}\Big), \tag{6}$$

and the corresponding eigenvectors are given by

$$\mathbf{v}_{\max}, \mathbf{v}_{\min} = \left[\frac{1}{2\xi k_2}\Big(k_1 + k_2 - \xi^2 k_2 \pm \sqrt{-4\xi^2 k_1 k_2 + (k_1 + k_2 + \xi^2 k_2)^2}\Big), 1\right], \tag{7}$$

Singular positions and configurations, in which a kinematic system gains an uncontrolled degree of freedom, are a special case of its stiffness in some direction going to zero. Typically, knowledge of such positions is used to design the usable workspace so as to avoid these singularities. For successful designs, the designer should consider Eqs. (1), (3), (4), and (6). The equations of motion of the variable rake angle mechanism and its numerical analysis can be found in [6]. In this prototype design, the important kinematic considerations are to accomplish a tool angle range of approximately -20 ~ +20 degrees and to limit the upper actuator stroke within 25.4 mm.

4. Robust repetitive controller design and experimental result

The controller for the variable rake angle mechanism is designed by two parameter robust repetitive control (TPRRC) design method [7]. The TPRRC design method has proved its superior performance compared to the conventional repetitive control based on zero-phase-error-tracking control (ZPETC) structure [8], in the sense that the TPRRC can incorporate low frequency disturbance rejection characteristics in its design and guarantee robust performance [9]. The discrete-time μ-synthesis block diagram for TPRRC design method is shown in Fig. 5, where z^{-1} represents the unit delay, N is the period of the periodic signal and L is the sum of the plant delay and the controller delay which comes from the inversion of the unstable zero part in the plant $G(z)$. The period of the periodic signal is much longer than the sampling rate, thus the z^{-N+L} has a very high order in general. The internal model of repetitive control is a periodic signal generator which can introduce an infinitely large feedback gain at the fundamental frequency and its harmonics. A (non-causal) zero-phase low-pass filter $q(z, z^{-1})$ is often used to improve the system stability by turning off the learning mechanism of the periodic signal generator in the high frequency range. The non-causal part of $q(z, z^{-1})$ may be absorbed in the adjacent long delay z^{-N+L}. $W_r(z)$ is an input

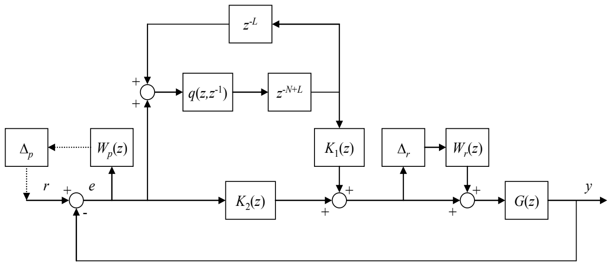


Fig. 5. Block diagram of TPRRC.

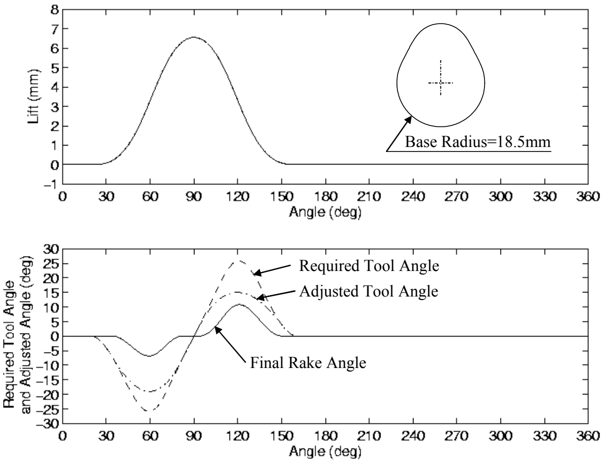


Fig. 6. Cam profile and adjusted tool angle.

multiplicative uncertainty and $W_p(z)$ is a performance weighting function. Δ_r and Δ_p are corresponding perturbations.

It is a well known fact that modern control design methods such as H_∞ control and μ-synthesis produce controllers of order at least equal to the plant, and usually higher because of the inclusion of weighting functions. With regards to computational complexity and practical implementation, the order of z^{-N+L} in the periodic signal generator is too high to be directly included in a discrete-time μ-synthesis formulation. A fictitious uncertainty Δ_f replacing z^{-N+L} is introduced and μ-synthesis technique is applied to design a robust repetitive controller $[K_1(z), K_2(z)]$, which is a two-input-one-output system. $K_1(z)$ is supposed to stabilize the positive feedback loop while $K_2(z)$ improves the robust performance of the closed-loop system.

A cam profile used in this research is shown in Fig. 6. If we want to keep 0 degree rake angle in the noncircular turning process with this cam profile, the tool angle should be changed -25.8~+25.8 degrees shown in the lower figure. The upper actuator used in this work is a voice coil actuator with a 25.4mm travel length, thus, due to the hardware limitations, the tool angle change is limited to -19.0~+15.0 degree range. As a result, the final rake angle is set to be -6.8~+10.8 degrees. The corresponding displacement curves for the cam profile and the final tool angle are shown in Fig. 7.

It is not easy to find a plant model quantifying the interaction between two actuator dynamics and we know that the

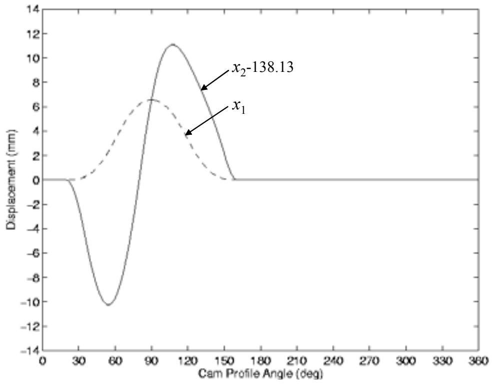


Fig. 7. Displacement curves.

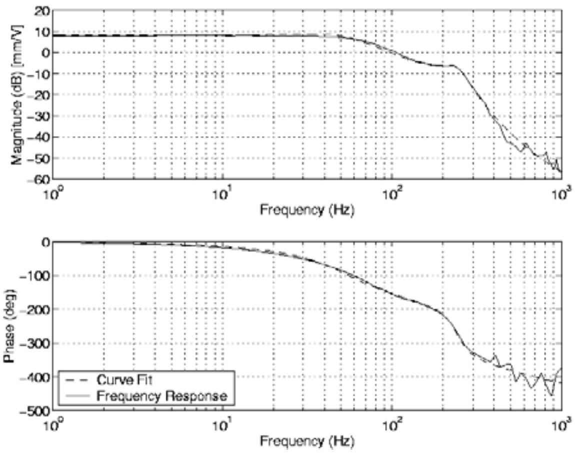


Fig. 8. Frequency response of the electrohydraulic actuator.

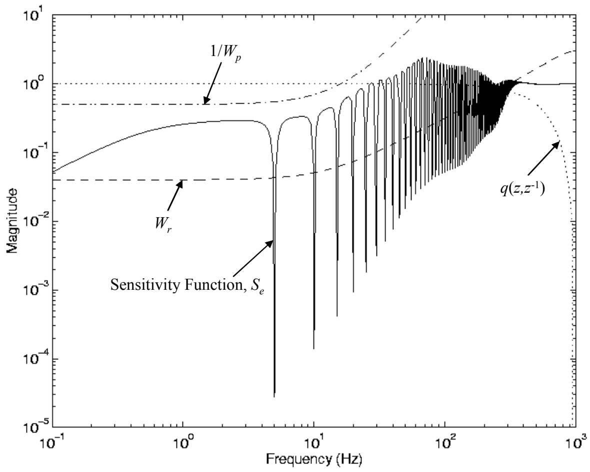


Fig. 9. Weights and sensitivity function for the electrohydraulic actuator.

most dominant dynamic effect is the inertia force in its own direction if we consider all moving elements. Thus, we used a SISO model for each actuator system by using frequency response tests and least squares fit. During the frequency response tests, the connecting rods (Body #3 in Fig. 4) were disconnected from the mechanism. Fig. 8 shows the frequency

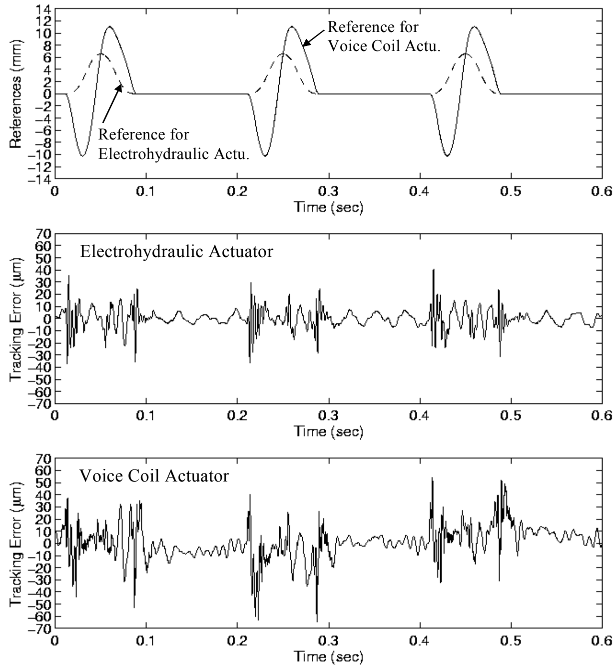


Fig. 10. Experimental tracking performance of the variable rake angle mechanism.

response of the electrohydraulic actuator and its least squares fit. With a sampling rate of 2 kHz, a 4th order discrete-time plant model of the electrohydraulic actuator was obtained from curve fit. Fig. 9 shows weighting functions and resultant sensitivity function from the designed robust repetitive controller for the electrohydraulic actuator. Similarly another repetitive controller was designed for the voice coil actuator.

We used a 2 kHz sampling rate and the references in Fig. 7 were fed to digitally implemented controllers at a spindle speed of 300 rpm, so the period of the periodic signal N was 400. The designed controllers were implemented on a TMS320C30 DSP board and two position feedback signals from the upper and lower actuators were collected by laser encoders with a 0.6 μm resolution.

The experimental results are shown in Fig. 10. The maximum tracking error of the electrohydraulic actuator was roughly 40 μm and it reaches up to 60 μm for the voice coil actuator. The rake angle error converted from two actuators' errors is less than 0.15 degrees, so it is negligible. The dynamic effects from the rotational tool part transfers to the electrohydraulic actuator side and it increases the tracking error. Since the final cam accuracy only depends on the lower actuator motion it is important to keep its tracking error as small as possible. One easy remedy to minimize the dynamic effect from the upper actuator to the lower actuator is changing the reference for the upper actuator. In this experiment the upper actuator reference was generated totally from geometric calculation. If it can be traded off for some smoothness such as continuity in the second derivative, then the upper actuator won't disturb the lower actuator that much. Alternatively, the upper actuator reference may be gradually changed to that of the lower actuator during final finishing process.

5. Conclusions

A prototype variable rake angle mechanism has been developed and controlled. The objective of the variable rake angle mechanism is to provide two-degrees-of-freedom to the conventional noncircular turning process. While the translational tool motion generates a noncircular profile, the rotational tool motion can compensate for the rake angle change caused by the noncircular profile itself. Inverse kinematics, kinematic sensitivity and stiffness, and design of the variable rake angle mechanism were discussed. The control system was designed from robust repetitive control design method and applied to the variable rake angle mechanism. Two actuators in the variable rake angle mechanism operated together to change the tool position as well as tool angle for a typical cam profile. Experimental results on the variable rake angle mechanism validate the design concept and control approach. Mechanical capabilities of the developed variable angle mechanism are under investigation and real machining results will be reported in the subsequent papers.

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